Magic Squares

In her interesting article "Constructing pandiagonal magic squares of arbitrarily large size" (*Mathematics Today*, February 2006), Kathleen Ollerenshaw on page 25 writes:-

"It is probable that no Su Doku solution can be Latin pandiagonal magic, but I have made no attempt to prove this".

Her surmise is correct, no 9x9 Su Doku solution can be Latin pandiagonal, because a Latin pandiagonal square of order 9 cannot exist. Inventing Latin squares, Euler more generally proved that **a pandiagonal Latin square cannot exist for 2k and 3k orders**. At the beginning of the 20th century, it was again proved by George Polya for the equivalent chess problem: a pandiagonal n-queens solution cannot exist for n=2k and 3k. In 1977, Hedayat proved again the non-existence of pandiagonal Latin squares of orders 2k and 3k in his paper on "Knut Vik" designs.

All this means that the smallest possible prime order of pandiagonal Latin squares is 5, as given in fig. 5 page 24. And the smallest possible composite orders of pandiagonal Latin squares are 25, 35, 49… Here is a pandiagonal Latin square of order 25, with additional characteristics of Sudokus in its sub-squares:

2	25	18	11	9	3	21	19	12	10	4	22	20	13	6	5	23	16	14	7	1	24	17	15	8
14	7	5	23	16	15	8	1	24	17	11	9	2	25	18	12	10	3	21	19	13	6	4	22	20
21	19	12	10	3	22	20	13	6	4	23	16	14	7	5	24	17	15	8	1	25	18	11	9	2
8	1	24	17	15	9	2	25	18	11	10	3	21	19	12	6	4	22	20	13	7	5	23	16	14
20	13	6	4	22	16	14	7	5	23	17	15	8	1	24	18	11	9	2	25	19	12	10	3	21
4	22	20	13	6	5	23	16	14	7	1	24	17	15	8	2	25	18	11	9	3	21	19	12	10
11	9	2	25	18	12	10	3	21	19	13	6	4	22	20	14	7	5	23	16	15	8	1	24	17
23	16	14	7	5	24	17	15	8	1	25	18	11	9	2	21	19	12	10	3	22	20	13	6	4
10	3	21	19	12	6	4	22	20	13	7	5	23	16	14	8	1	24	17	15	9	2	25	18	11
17	15	8	1	24	18	11	9	2	25	19	12	10	3	21	20	13	6	4	22	16	14	7	5	23
1	24	17	15	8	2	25	18	11	9	3	21	19	12	10	4	22	20	13	6	5	23	16	14	7
13	6	4	22	20	14	7	5	23	16	15	8	1	24	17	11	9	2	25	18	12	10	3	21	19
25	18	11	9	2	21	19	12	10	3	22	20	13	6	4	23	16	14	7	5	24	17	15	8	1
7	5	23	16	14	8	1	24	17	15	9	2	25	18	11	10	3	21	19	12	6	4	22	20	13
19	12	10	3	21	20	13	6	4	22	16	14	7	5	23	17	15	8	1	24	18	11	9	2	25
3	21	19	12	10	4	22	20	13	6	5	23	16	14	7	1	24	17	15	8	2	25	18	11	9
15	8	1	24	17	11	9	2	25	18	12	10	3	21	19	13	6	4	22	20	14	7	5	23	16
22	20	13	6	4	23	16	14	7	5	24	17	15	8	1	25	18	11	9	2	21	19	12	10	3
9	2	25	18	11	10	3	21	19	12	6	4	22	20	13	7	5	23	16	14	8	1	24	17	15
16	14	7	5	23	17	15	8	1	24	18	11	9	2	25	19	12	10	3	21	20	13	6	4	22
5	23	16	14	7	1	24	17	15	8	2	25	18	11	9	3	21	19	12	10	4	22	20	13	6
12	10	3	21	19	13	6	4	22	20	14	7	5	23	16	15	8	1	24	17	11	9	2	25	18
24	17	15	8	1	25	18	11	9	2	21	19	12	10	3	22	20	13	6	4	23	16	14	7	5
6	4	22	20	13	7	5	23	16	14	8	1	24	17	15	9	2	25	18	11	10	3	21	19	12
18	11	9	2	25	19	12	10	3	21	20	13	6	4	22	16	14	7	5	23	17	15	8	1	24

This 25x25 Sudoku is a pandiagonal Latin square: each sub-square, row, column, diagonal, broken diagonal contain all numbers from 1 to 25. It is *impossible* to construct a pandiagonal Latin square of a smaller composite order (including the impossibility of a 9x9 pandiagonal Latin

We make two other remarks directly linked to the Ollerenshaw's article:

- Bimagic squares have the nice supplemental feature of remaining magic after squaring their numbers. Thus Bimagic squares can be constructed using two Sudokus.

11	77	35	46	4	70	57	42	27
55	40	25	12	78	36	47	5	71
48	6	72	56	41	26	10	76	34
80	29	14	7	64	49	45	21	60
43	19	58	81	30	15	8	65	50
9	66	51	44	20	59	79	28	13
32	17	74	67	52	1	24	63	39
22	61	37	33	18	75	68	53	2
69	54	3	23	62	38	31	16	73

A 9x9 bimagic square. Magic sum of rows, columns, diagonals: 369. After squaring the numbers ,the magic sum is 20,049. The 9 cells of each 3x3 sub-square have the same magic sums.

- Multiplicative squares are the squares which are magic using multiplication of their numbers, instead of addition. Pandiagonal multiplicative magic squares can be constructed using pandiagonal Latin squares.

1	10	21	32	54
28	48	9	2	15
18	3	20	42	8
30	7	16	27	4
24	36	6	5	14

A 5x5 pandiagonal multiplicative square. When you *multiply* the 5 numbers of any row, any column, any diagonal, any broken diagonal, you always get 362,880.

On these remarks, and others, you will find details in my website.

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C. Boyer, Some notes on the magic squares of squares problem, *The Mathematical Intelligencer*, Vol 27, Number 2, 52-64 (Spring 2005)

J. Dénes and A.D. Keedwell, *Latin Squares and Their Applications*, page 214, Akadémiai Kiado, Budapest (1974)

L. Euler, Recherches sur une nouvelle espèce de quarrés magiques, *Verhandelingen uitgegeven door het zeeuwsch Genootschap der Wetenschappen te Vlissingen* 9, 85-239 (1782), reprint in *Euleri Opera Omnia*, Series I, Vol 7, 291-392 (1923)

A. Hedayat, A complete solution to the existence and non-existence of Knut Vik designs and orthogonal Knut Vik designs, *Journal of Combinatorial Theory* (A) 22, 331-337 (1977)

G. Polya, Über die doppelt-periodischen Lösungen des n-Damen-Problems, in W. Ahrens, *Mathematisch Unterhaltungen und Spiele*, Vol 2, pages 364-374, Teubner, 2nd edition, Berlin (1918)

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